EXAMINING FREQUENCY-DISPLACEMENT RELATIONSHIP OF PILES IN FINE GRAINED SOIL

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Abstract- This research work focuses on the dynamic response of pile foundations subjected to lateral loading in fine-grained soil by employing Winkler Beam-Spring Foundation model (WBSF) with a Reduced-Dimension Finite Element Model (RDFEM). Three pile diameters (0.76m, 1.2m, 2.0m) were selected for the analysis. The results noted that the WBSF model consistently predicted 20-30% greater displacements and lower natural frequencies) due to its simplified spring-based soil representation. On the contrary, the RDFEM model predicts higher excitation frequencies and reduced displacements as it corporates 3D soil continuity, yielding more realistic soil-pile dynamic interaction. These findings align with already published research, demonstrating that WBSF serves for preliminary analysis, and its limitations in capturing true soil-pile interaction make it unsuitable for critical dynamic loading cases. The RDFEM gives higher accuracy, particularly in simulating seismic effects and soil yielding, provides essential insights for safe foundation design. This work highlights the necessity of advanced modeling techniques for reliable prediction of pile behavior in fine-grained soils subjected to lateral loads, offering practical guidance for geotechnical engineering practice.

Keywords- Pile Foundation, Dynamic Loading, Winkler Method, Pile Response Spectrum

1 Introduction

Pile foundations can be categorized in numerous ways such as by material, construction method, or structural function. However, in geotechnical engineering, the most critical classification is based on how piles behave under certain loads. Generally, piles fall into two main categories. Piles that simply rely on skin friction along their longitudinal direction to support loads are defined as friction piles whereas piles transfer most of the load to a hard stratum at the pile tip, resisting settlement through base reaction is termed as end bearing piles. The load-settlement behavior and axial load transfer mechanism of piles have been extensively studied. However, the response of piles subjected to lateral loading, particularly dynamic lateral loading (particularly generated by earthquakes), remains a complex and less understood phenomenon, due to very limited research on dynamic behavior of pile subjected to lateral load. This lead to an opportunity to explore this area in depth, to mitigate the risk of catastrophic failure in poorly designed foundations, a comprehensive understanding of pile-soil interaction under lateral loading is essential for ensuring structural safety and operational efficiency in geotechnical engineering

According to existing research, lateral load analysis using numerical techniques can generally be grouped into three primary categories: (i) the Winkler-based approach, (ii) analytical or closed-form models, and (iii) the

Finite Element Method (FEM). In the Winkler approach, the soil's reaction is modeled through discrete springs, while the pile is considered as a linearly elastic beam. This concept, first introduced in 1967, linked dynamic soil stiffness with spring elements, laying the groundwork for further developments in this field [1]. Later enhancements included the application of harmonic loads, resulting in mathematical models that estimated displacements at the pile head [2-4]. A significant advancement was made by Nogami and colleagues, who introduced nonlinear soil behavior through the development of p–y curves [5]. With improvements in computing capabilities, FEM became a popular tool for analyzing pile behavior under lateral loads. Soil was represented using various element types, including 8-node bricks and triangular meshes [6-9]. Laboratory experiments were also carried out to validate these FEM-based models [10-12]. However, FEM is often limited by its substantial computational demands. To address this, a simplified approach—Reduced-Dimension FEM (RDFEM)—was introduced, which excludes vertical displacements of both soil and pile, as these have minimal effect on lateral behavior [13]. Subsequent experimental validation of RDFEM confirmed its reliability, offering researchers a refined method for predicting the response of piles subjected to lateral loading[14]. Based upon the already published research, the study utilized two famous methods WBSF and RDFEM for investigating the pile response subjected to lateral loading in fine grained soil. The detailed methodology and results are reported in the proceeding sections

2 Research Methodology

2.1 Data Collection and Selection of Parameters:

For investigation the data was collected from 56 boreholes distributed over the Rawalpindi-Islamabad region to study soil characteristics for supporting numerical modeling. The soil strata were classified into distinct layers, extending 50 meters deep, based on corrected Standard Penetration Test (SPT-N) results. From these values, crucial parameters like Vs, Vp, unit weight, elastic modulus, Poisson's ratio and shear modulus (G) were estimated using well established empirical co-relations related to fine grained soil [15-17].

2.2 Numerical Modelling Using Winkler Beam Spring Foundation Model

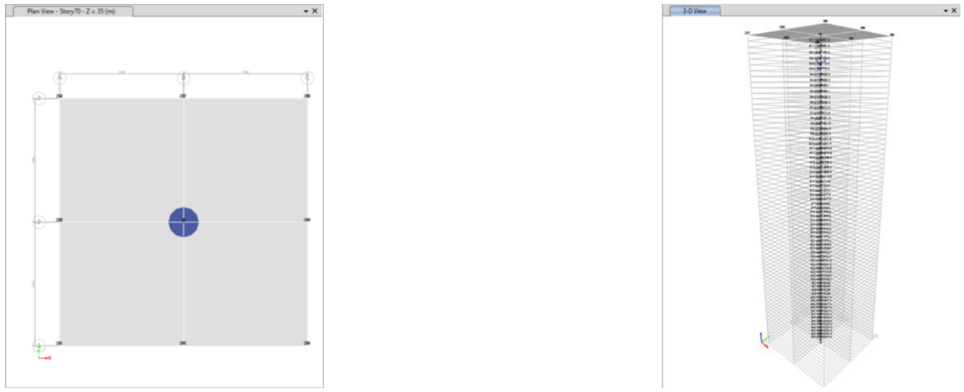
The Winkler Beam-Spring Foundation (WBSF) model is a popular method used to analyze the behavior of piles subjected to lateral load. It reduces the complexity of soil-structure interaction by modeling the soil as a series of independent linear springs. In this case, the reaction from the soil (p) at any point is proportional to the pile's displacement laterally (y), k is modulus of subgrade reaction (force/length), mathematically:

$$p = ky \tag{1}$$

The development of WBSF model includes two main steps. First, is determination of the dynamic spring constant for each individual soil layer, and second is numerical simulation of the soil-pile system. For the first step, a code in MATLAB was written to compute the dynamic stiffness spring constant of soil layers including both components of soil resistance, real (spring stiffness) and imaginary (damping) in the WBSF, following the framework proposed in published study [3]. Next, a numerical model was developed in ETABS to simulate the dynamic response of a pile foundation embedded in layered fine grained. Beam element is employed to represent pile whereas 1D spring element is used for soil. A superstructure is represented by modeling a high pile cap using area elements. WBSF model of pile cap and pile is shown in Figure 1 (a) and (b) respectively.

2.3 Numerical modelling using Reduced Dimension Finite Element Model

A computational efficient RDFEM approach was adopted, where the soil was represented as continues ring elements with radial, azimuthal, and axial displacement degrees of freedom (DOFs). The pile was modeled with beam elements whereas pile cap was modelled using area element. The soil boundaries and mesh size was selected on the basis of soil dynamic characteristics



(a) *(b)*

Figure 1: WBSF Model of Pile Cap and Pile

3 Results and Discussion:

Frequency displacement relationship of all three piles having diameter of 0.76m, 1.2m and 2.0m is presented in Figure 2, 3 and 4. The pile response spectrum for the 0.76 diameter pile was investigated at frequencies ranging from 1 Hz to 10 Hz. The WBSF method indicated natural frequency of 1 Hz with corresponding displacement of 1.87x10-4 m. The FEM analysis, however, predicted a slightly higher natural frequency of 1.5 Hz and a smaller displacement of 1.47×10^{-4} meters

Due to its ability to capture complex soil-pile interaction behavior, FEM generally produces more results that are accurate. For pile diameter 1.2m. The WBSF method indicated natural frequency of 1.5 Hz with corresponding displacement of 1.87894×10^{-5} m. However, a slightly smaller displacement of 1.51×10^{-5} meters and a higher natural frequency of 2 Hz were found in the FEM analysis. For pile diameter 2.0m, The WBSF method indicated natural frequency of 2.0 Hz with corresponding displacement of 5.37×10^{-6} m. However, a slightly smaller displacement of 4.05×10^{-6} meters and a higher natural frequency of 2.5 Hz were found in the FEM analysis. In particular, the natural frequencies predicted by WBSF were 1 Hz, 1.5 Hz, and 2 Hz for each increasing diameter while FEM provided results of 1.5 Hz, 2 Hz and 2.5 Hz. Likewise, displacements calculated from WBSF were about twenty to thirty percent more than those estimated using FEM. This suggests that the simplified WBSF method exaggerates flexibility whilst underestimating stiffness in soil-pile systems. Due to oversimplification and breaking continuum nature of soil, unlike FEM which captures the true complexity of soil behavior. The comparison of the obtained frequency and displacement values from FEM and WBSF model is presented in Table 1.

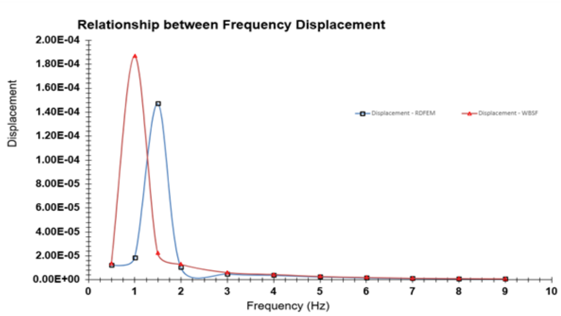


Figure 2: Frequency –Displacement Relationship (Dia. 0.76m)

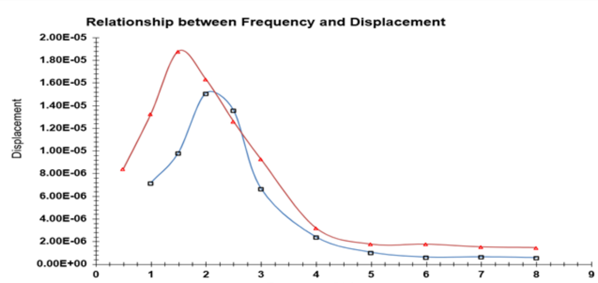


Figure 3: Frequency –Displacement Relationship (Dia. 1.2m)

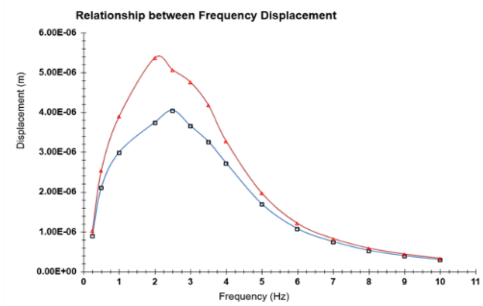


Figure 4: Frequency –Displacement Relationship (Dia. 2.0m)

Table 1: Natural frequencies and Displacement

Diameter	RDFEM		WBSF		% Difference
	Natural frequency	**Displacement**	**Natural frequency**	**Displacement**	
0.76	1.5	1.47E-04	1	1.87E-04	21.39
1.2	2	1.51E-05	1.5	1.87E-05	19.63
2.0	2.5	4.05E-06	2.0	5.37E-06	24.58

The increase in natural frequencies and the corresponding decrease in displacement due to increase in pile size is primarily due to increase in the stiffness of the system (Pile-Soil combined system). The increase in pile size increases its flexural rigidity, which enhanced the stiffness. It is a well-established fact that stiffness governs the overall dynamic response of the pile. The same observation is confirmed in the results of the present study. The inaccurate estimation of dynamic response by WBSF is primarily attributed to its simplification adopted in modeling the soil. As stated above, 1D springs are used in WBSF model to denote soil, which in reality is a 3D continuum material. By doing so, the shear interaction among the soil layers is ignored in WBSF which resulted in higher displacement values along with lower fundamental frequencies. On the contrary, the whole 3D continuum nature of soil can be modelled in FEM. As a result, shear interaction among the soil layers is considered in FEM analysis which helps to produce more accurate results, and the model is close to realistic representation of the soil.

The present study provides useful insights in the dynamic soil-pile characteristics for Islamabad Rawalpindi region. However, the present study is based on the use of empirical co-relation to determine dynamic properties of the soil. Authors recommend examining the pile-soil dynamic behavior by using the actual soil dynamic properties obtained from field experiments to enhance the reliability of the results.

4 Conclusion

The following conclusions are drawn from the present study:

1. Maximum amplitude of displacement, bending moment and shear force are observed at the top of pile at the intersection point of pile and pile cap.

2. WBSF model overestimates the lateral response of piles whereas FEM provides realistic results.

3. The overestimation by WBSF is primarily due to simplification adopted in modeling the soil breaking the continuum nature of the soil.

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