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STABILITY ANALYSIS OF SLOPES USING LIMIT EQUILIBRIUM AND FINITE ELEMENT METHODS

[a*]Sohaib Naseer, [b]Robert Evans

[a &b] School of Architectural, Design and Built Environment, Nottingham Trent University, UK
[a*]sohaib.naseer2019@my.ntu.ac.uk

Abstract- The stability of natural and manmade slopes is a common geotechnical problem. Due to its importance, the analysis of slope stability has received wide attention in literature. Various methods have been developed to analyze slope stability each of which is based on different assumptions and conditions. Each technique has some advantages and limitations. This paper compares the factor of safety and failure surfaces of slopes obtained by limit equilibrium and finite element method. For this purpose, Rocscience SLIDE 6.0 for limit equilibrium (LE) and PLAXIS for finite element (FE) analysis was used. The safety factors were calculated for different slope geometry and soil types. A comparison was made among the slip surfaces and factor of safety values obtained using both approaches. It was concluded that LE approach, estimated a higher factor of safety as compared to the FE approach. Also, significant variation is found is the failure surfaces as determined from both approaches.

Keywords- Slope Stability, Limit Equilibrium, Finite Element Method, PLAXIS, Rocscience Slide 6.0

1 INTRODUCTION

Landslides or slope failures are the major threats to both human lives as well as property. The adequate assessment of slopes is a major concern of geotechnical engineers that how close or far the slopes are from failure. To investigate the factor of safety for slope, various methods have been introduced by the researchers. The earliest studies appeared in the 1970s (e.g., [1]; [2]; [3]; [4]) and have continued steadily (e.g., [5]; [6]; [7]; [8] [9]). However, in recent years, due to the development in the field of computational methods, various software programs have been developed for the slope stability analysis ([10]; [11]; [12]). The stability methods are commonly categorized in two major groups. The one is Limit Equilibrium Methods (LEM) and other is numerical analysis. The LEM is based on creating the balance conditions of driving and resisting forces acting on a slope. The slope will be in stable condition when the resisting forces (internal forces of slopes) are equal to more than the driving forces (external forces applied on slope including the self-weight of slope). Numerical analysis is based on dividing the slopes in finite number of elements and zones. Afterward, forces and displacement/strains are calculated using the constitutive laws in the slope. The numerical analysis methods are further divided into different techniques which included Finite Element Methods (FEM), Discrete Element Methods (DEM) and Boundary Element Methods (BEM) [13]. In this paper, the analysis of slope stability was carried out using limit equilibrium method and finite element method. The factor of safety obtained from these methods are then compared. Furthermore, the mode of slope failure is investigated using both methods, and effect of surcharge on stability of slope was also examined.

2 LIMIT EQUILIBRIUM METHOD

The analysis of slopes using the Limit Equilibrium Method (LEM) has been significantly refined by using various methods of vertical slices. It is based on the most common five limiting equilibrium methods for determining the safety factor, i.e., ordinary or Fellenius method [14], Bishop's simplified method [15], Janbu's simplified method [16], Spencer's method [17] and the Generalized Limiting Equilibrium (GLE) or Morgenstern–Price method [18]. Limit Equilibrium (LE) methods use the Mohr-Coulomb failure criterion to determine the shear strength along the slip surface. A state of limit equilibrium exists when the mobilized shear stress is expressed as a fraction of the shear strength. In LE analysis, the sliding mass is divided into slices, determination of the shear and normal inter-slice forces is made, and appropriate force and/or moment equilibrium equations are satisfied for static equilibrium conditions. The first LE method for a round slip surface was presented by Fellenius [14]. Bishop [15] later developed a revised method of circular slip analysis. Meanwhile, Janbu [16] presented a technique for non-circular failure surfaces that isolated a potential sliding mass into a few vertical slices. Later techniques were developed by Morgenstern-Price [18], Spencer [17], Sarma [19] and a few others to make further advances with regards to the various assumptions about inter-slice forces. The LE methods chosen for this study included Bishop's Simplified Method (BSM), Janbu's Simplified Method (JSM), and Spencer's Method (SM). These methods are commonly used due to relatively adequate accuracy while calculating the FOS.

3 FINITE ELEMENT METHOD

Numerical modelling is considered a dominant tool to solve the complex engineering problems and has become increasingly popular in geotechnical engineering analysis. The two most common types of numerical methods are Finite Element (FE) and Finite Different (FD) methods. The finite element (FE) method, discussed in this paper, uses the soil stress-strain behaviour for slope stability modelling. The major factor which considered as advantage of finite element analysis is that no assumptions are required for the slip surface and shape of the slope. The FE approach divides the model into a number of pieces or elements of a mesh. Stresses and strains are calculated using the constitutive laws for materials comprising of the slope stability model. Failure occurs naturally through the zones in which the soil shear strength is unable to sustain the developed shear stresses. Ultimately, a Reduction Factor (RF) can be calculated for finite element methods using the 'c-φ reduction' method. This approach requires incrementally reduced soil strength parameters until the failure occurs. The shear strength reduction technique enables the FE method to calculate FOS for slope.

4 SLOPE MODEL

4.1 Slope Geometry

It is well understood that stability of slope is a direct function of the height and gradient of slope, as well as material properties of the soil/rock material. For purposes of this study, a simple slope section was modelled with combinations of several different slope heights and gradients with a variety of different soil types. Four different slope gradients (β) of 1H:1V, 1.5H:1V, 1.75H:1V & 2H:1V were modelled at three different heights (H) of 7m, 12m & 17m, resulting in a total of twelve (12) different combinations of slope geometry. The base of the slope model was assumed to be rigid (or bedrock).

4.2 Soil properties

Four different soil types with unique assumptive soil properties were modelled as homogenous materials for slope stability analyses using both LE and FE methods. The selection of these soil types was intended to provide a wide array of strength characteristics ranging from cohesionless to purely cohesive soils, modelled as loose/soft and dense/hard. Table 1 provides a summary of the parameters used to create the various soil types for purposes of this study.

Table 1: Soil types and assumed elastic/plastic parameters for study

Sr. No.	Soil Type	Soil ID	μ	E (MPa)	ɣ (kN/m^3)	c (kPa)	Φ (degree)
1	Soft Sandy Clay (SC)	S_1	0.30	25	16.50	20	21
2	Stiff Sandy Clay (SC)	S_2	0.25	100	18.00	50	24
3	Soft Clay (CL)	S_3	0.4	18	16.00	10	0
4	Stiff Clay (CL)	S_4	0.35	40	17.00	30	0

5 ANALYSIS APPROACHES

5.1 Limit Equilibrium Analysis

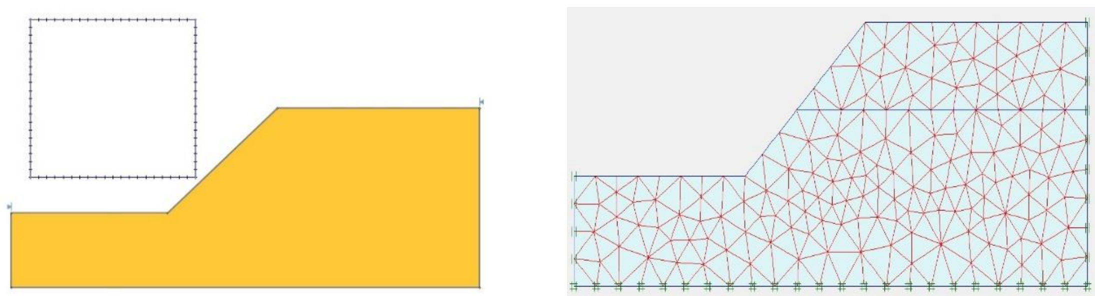
Limit equilibrium (LE) analysis was performed by the commercially available LE tool SLIDE software, advanced by Rocscience, Inc. It is a two-dimensional computer program, which can be used to evaluate the stability of circular or non-circular failure surfaces. SLIDE is widely used in the engineering world for relatively quick and easy computations of a variety of different slope configurations. Computations of FOS in SLIDE were based on the Mohr-Coulomb failure criterion without tension cracks. The model requires basic strength parameters such as the angle of internal friction (φ) and cohesion (c). Figure 1(a) shows the typical auto grid generated for determination of critical slip surface in Slide.

5.2 Finite Element Analysis

Commercially available finite element tool, PLAXIS is used for two dimensional FE analysis. The safety factor in PLAXIS is determined using strength reduction method, also known as ϕ-c reduction method. In this approach, shear strength parameters (ϕ and c) of the soils are successively reduced until failure occurs. PLAXIS uses a total multiplier parameter ΣM_{sf} to define the value of the shear strength parameters at a given stage and calculates the safety factor (SF) from the value of ΣM_{sf} which is equal to the available strength divided by the strength at failure:

Safety Factor= (resisting strength)/ (driving strength) =$\sum M_{sf}$ at ***failure*** ------------------- Equation (1)

Figure 1(b) shows the typical mesh generated for determination of finite element analysis in PLAXIS.

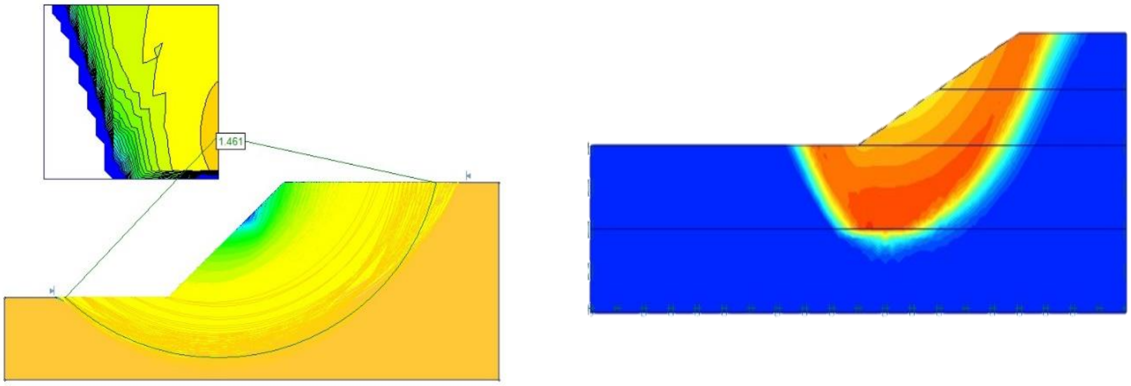


(a) (b)

Figure 1: (a) Typical auto-grid generated by Slide for slip surface. (b) typical mesh generated using PLAXIS 2D

6 RESULTS AND DISCUSSIONS

The slope stability analysis were performed on 4 different soils with 4 slope angles with 3 different slope heights using LEM and FEM method. BSM, JSM and GLE methods were used to determine the safety factor in LEM, while in FEM, strength reduction method was adopted for calculation of safety factor. The results are summarized in Table 2 – Table 5. Figure 2 illustrate the critical failure surface developed in slide and PLAXIS analysis. Figure 3 shows the comparison of safety factor determined using different approaches in LEM and FEM for various geometrical and soil conditions. It was observed that LE overestimate the safety factors as compared to FE. LE estimated the factors of safety about 20% -30% more than the FE in all cases. Also, among the different approaches used in LE, the BSM calculated the higher values as compared to JSM. The reasons for this variations are the analysis mechanism of all LE methods. Every method have some assumptions, e.g. BSM did not consider the vertical forces between the slices, whereas, in JSM and GLE methods, the inter-slices vertical forces and individual slice momenta are considered which estimate the higher value of safety factors. Furthermore, it can be seen that, stiff soils shown more resistance towards the failure and hence gave higher values of safety factors as compared to the soft soils. The stiff soils have more tendency to resist the applied forces and hence produces more resistance at the failure surfaces because of higher shear strength characteristics. Among the all soil types, the stiff sandy clay (S_B) shown the maximum resistance whereas, soft clay (S_D) gave the minimum values of safety factor in all geometrical conditions. Also, factor of safety decreases for steep slopes in both LE and FE analysis. When the height of slope is increased, there is significant drop in the factor of safety in both approaches.



a) b)

Figure 2: Typical slip surface developed using (a) LEM and (b) FEM

Table 2. Calculated FOS for critical surface using LEM and FEM for slope = 1H: 1V

Soil Type	Slope Height = 7 m				Slope Height = 12 m				Slope Height = 17 m			
	LEM			FEM	LEM			FEM	LEM			FEM
	BSM	JSM	GLE		BSM	JSM	GLE		BSM	JSM	GLE	
S_1	1.80	1.79	1.82	1.12	1.33	1.28	1.32	0.98	1.12	1.07	1.12	0.87
S_2	3.29	3.26	3.29	2.50	2.26	2.22	2.26	1.75	1.84	1.78	1.83	1.00
S_3	0.52	0.51	0.52	0.10	0.32	0.35	0.37	0.10	0.22	0.23	0.23	0.10
S_4	1.46	1.45	1.46	0.90	0.91	0.99	1.05	0.65	0.62	0.65	0.66	0.30

Table 3. Calculated FOS for critical surface using LEM and FEM for slope = 1.5H:1V

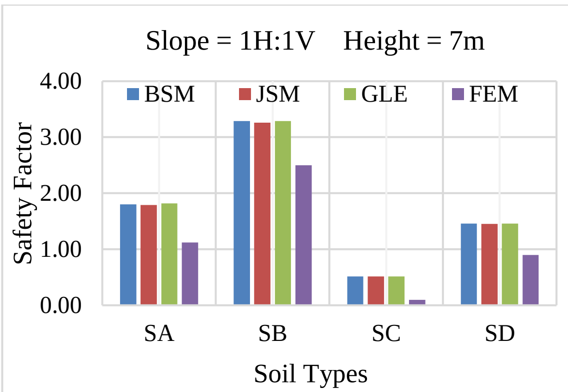
Soil Type	Slope Height = 7 m				Slope Height = 12 m				Slope Height = 17 m			
	LEM			FEM	LEM			FEM	LEM			FEM
	BSM	JSM	GLE		BSM	JSM	GLE		BSM	JSM	GLE	
S_1	1.97	1.89	1.96	1.35	2.82	2.54	2.82	1.95	1.40	1.29	1.39	0.90
S_2	3.52	3.45	3.52	2.20	4.79	4.31	4.78	3.20	2.23	2.08	2.23	1.65
S_3	0.52	0.51	0.52	0.20	0.66	0.63	0.65	0.30	0.27	0.29	0.28	0.15
S_4	1.46	1.43	1.46	1.00	1.85	1.79	1.86	1.20	0.75	0.81	0.81	0.50

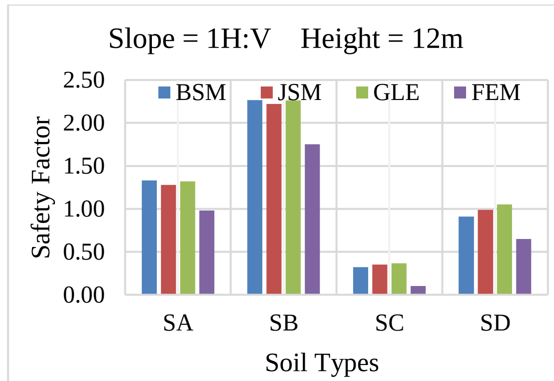
Table 4. Calculated FOS for critical surface using LEM and FEM for slope = 1.75H:1V

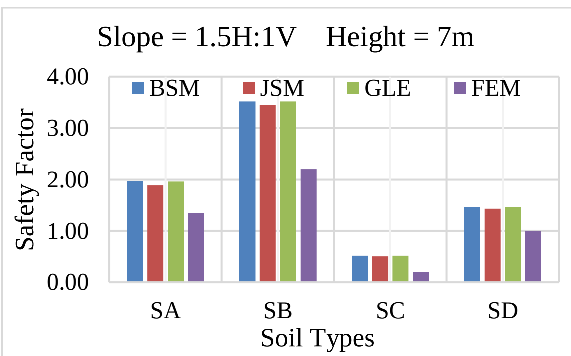
Soil Type	Slope Height = 7 m				Slope Height = 12 m				Slope Height = 17 m			
	LEM			FEM	LEM			FEM	LEM			FEM
	BSM	JSM	GLE		BSM	JSM	GLE		BSM	JSM	GLE	
S_1	2.34	2.17	2.33	1.75	3.30	2.94	3.29	2.10	1.60	1.47	1.60	0.75
S_2	4.08	3.80	4.08	2.75	5.59	5.01	5.58	3.80	2.59	2.41	2.58	1.85
S_3	0.55	0.53	0.55	0.25	0.78	0.74	0.78	0.35	0.33	0.35	0.35	0.20
S_4	1.56	1.51	1.56	1.10	2.20	2.10	2.20	1.50	0.92	0.99	1.00	0.60

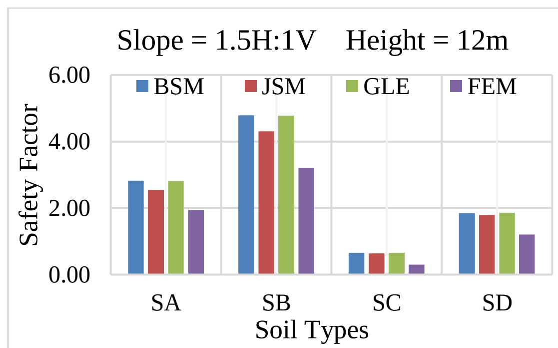
Table 5. Calculated FOS for critical surface using LEM and FEM for slope = 2H: 1V

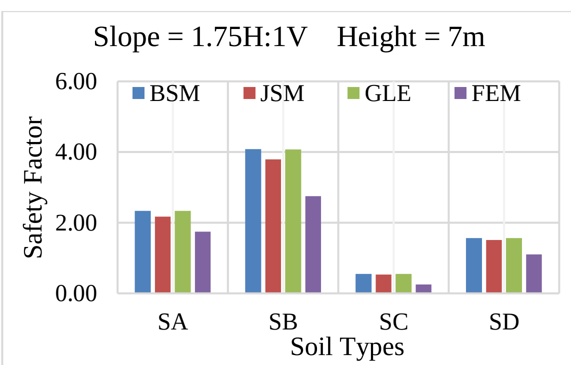
Soil Type	Slope Height = 7 m				Slope Height = 12 m				Slope Height = 17 m			
	LEM			FEM	LEM			FEM	LEM			FEM
	BSM	JSM	GLE		BSM	JSM	GLE		BSM	JSM	GLE	
S_1	2.48	2.29	2.47	1.80	3.24	2.92	3.23	2.20	1.64	1.51	1.63	0.80
S_2	1.12	1.07	1.12	0.87	5.22	4.68	5.21	4.10	2.54	2.32	2.54	1.90
S_3	0.58	0.55	0.58	0.30	0.63	0.59	0.63	0.30	0.27	0.27	0.27	0.15
S_4	1.63	1.56	1.63	1.20	1.79	1.67	1.79	1.30	0.76	0.75	0.75	0.55

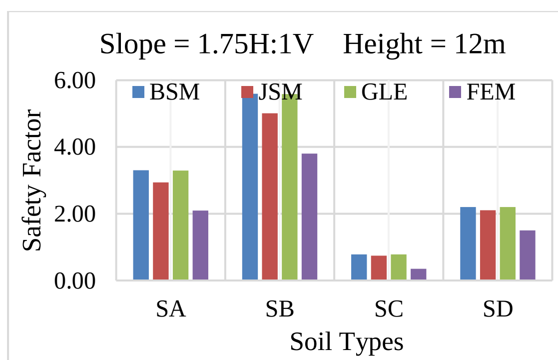












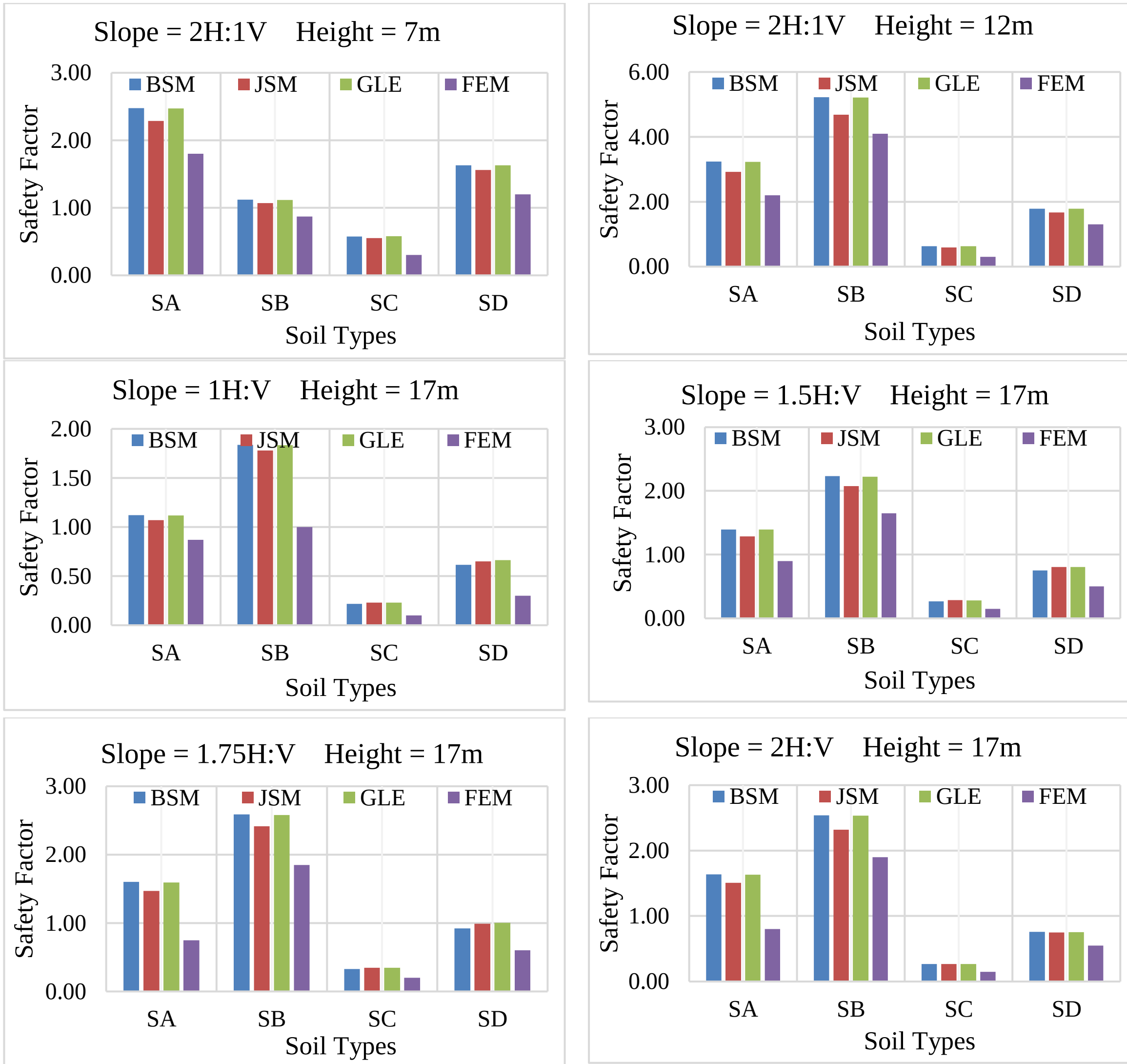


Figure 3: Comparison of safety factors calculated using LEM and FEM for different slope geometry and soil types

7 CONCLUSION

In general, LE methods are used for slope stability analysis by the practicing engineers and professionals. These methods provide rationally reliable values for safety factors, but LE methods are lacked in stress-strain behaviour and thus are not suitable for realistic analysis of complex slopes. This issue has been resolved by FE techniques for analysis which are based on fundamental principles of stress-strain behaviour. Both approaches have certain advantages and limitations and can be used for determination of slip surface and safety factors for slope. However, the similar studies have confirmed the vigorous nature of FE approach. Based on the current study, following conclusion are made:

- In general, the LE and FE methods used in this study provide fairly consistent FOS. When comparing relatively similar critical failure surfaces between the LE and FE methods, FE analyses show higher concentrations of plastic strain near the toe of the slope.
- The critical slip surface developed in FE is near the slope face, but in LE, the critical slip surface is found at far away from the slope face.
- The FE consider the interface strength which affects the factor of safety, whereas, the LE does not consider the interface strength that ultimately depicts in the calculated values of safety factor as shown in above sections.

- Although slope geometry plays a significant role in determining safety factors for stability of slopes, soil shear strength properties dominate as the primary-most factor in computing safety factors and producing unique failure surfaces.
- The stability of slopes in less cohesive soils is controlled by shallow surficial failures. Hence, slope gradient (β) is the primary factor in determining safety factors for cohesionless soils. Relatively similar critical failure surfaces were noted between the LE and FE methods for cohesionless sands. Slope stability analyses of purely cohesive soils result in deep-seated critical failure surfaces. Slope height (H) was observed to be the primary aspect of slope geometry affecting slope stability of clayey soils. FOS computed for these soils were notably higher than the equivalent RF. The LE method resulted in a relatively defined critical failure surface, whereas the FE method showed a much wider zone of critical failure.

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