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A DIFFUSE DAMAGE MODEL FOR ASPHALT CONCRETE

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Abstract- Distress in a pavement is a serious problem which can reduce the service life of a pavement. For an efficient design and analysis of pavements, use of efficient yet accurate computational models play a key role. Several numerical models have been used in the past to understand the mechanics of cracking in asphalt concrete. Interface elements with cohesive zone model have been successfully employed by many researchers however, the method requires the crack path to be known a priori and the cracks can only grow along element boundaries. On the contrary, continuum damage/ plasticity models offer the ease of damage modeling, but these methods show mesh dependency. In this paper a phase field diffuse damage model integrated with cohesive zone concept is used to simulate damage in asphalt concrete. The proposed model can simulate multiple interacting cracks propagating arbitrarily through the finite element mesh without the need of any *ad hoc* criterion. The effectiveness of the model is demonstrated using a single edge notch beam. Numerical results are validated against experimental observations. The obtained load versus crack mouth opening displacement curve quantitatively and damage profile qualitatively show good agreement with the experimental observations. The proposed model successfully simulated sharp crack and does not suffer from mesh dependency problem. Additionally, the model is also able to overcome the issue of complaint material behavior before cracking due to the presence of dummy stiffness in the interface element formulations.

Keywords- Asphalt concrete, finite element method, phase field method, mode-I fracture.

1 Introduction

Distress in asphalt pavements can severely affect its performance. Cracking in pavements not only increases the maintenance cost but also can affect the durability and life span of the pavement. Therefore, it is pivotal to understand the damage mechanics of asphalt mixtures for an efficient design. Numerical analysis of materials and structures plays a key role in the design process and is useful to simulate damage under various boundary conditions.

A finite element method with interface elements is often used to simulate fracture in asphalt mixtures. Soares et al. [1] used interface elements with cohesive zone model to simulate fracture in an indirect tension test specimen. Song et al. [2] used interface elements with exponential cohesive constitutive law to simulate mode-I cracking in asphalt concrete beam. Dave et al. [3] used the cohesive zone model to simulate cracking in asphalt mixtures due to thermal loads. Due to the restriction of a crack to propagate along element boundaries in the interface element model, mesh independent crack growth methods like extended finite element method (XFEM) [4] is also explored by many researchers. Mahmoud et al. [5] and Islam et al. [6] used extended finite element method with cohesive zone model to simulate fracture in asphalt mixtures. Even though, XFEM is a good method to simulate mesh independent cracking, modelling complex crack typologies is still a challenge. Moreover, for the case of multiple interacting cracks the method becomes cumbersome from a computer implementation point of view. On the contrary, diffuse damage models were also used by some researchers. Park et al. [7] presented a viscoelastic damage model for the asphalt mixture. Chehab [8] proposed a viscoelastic-plastic approach for damage in asphalt mixtures. However, these methods suffer from mesh dependency problems. As the mesh is refined the dissipated energy approaches zero.

This paper presents a phase field model to simulate damage in asphalt mixtures. In the phase field method, a solid material is divided into damage and undamaged phases using a damage phase field variable. A free energy functional/crack potential is constructed based on the damage phase field variable and the whole system evolves towards the direction which minimizes this potential. The phase field model can efficiently simulate crack nucleation, propagation and complex crack typologies without the need of any crack tracking algorithm and with a simple computer implementation unlike interface element formulations. Moreover, the method does not suffer from the mesh dependency problem as observed in the continuum damage formulations. Bourdin et al. [9] used a variational approach to regularize the crack potential. The pioneering work of Bourdin et al. [9] on modelling brittle fracture in solids using phase field model has led to many advancements in this model. Miehe et al. [10] and Miehe et al. [11] presented a phase field model using thermodynamic considerations for damage modelling in brittle materials. Recently, Wu [12] extended the phase field model to simulate brittle and quasi-brittle fracture in solids by integrating a cohesive zone model with the phase field model.

In this paper the phase field model of Wu [12] is used to simulate quasi brittle fracture in asphalt concrete. To the best of authors knowledge, very little work has been done on simulating damage in asphalt mixtures using the phase field model. It is worth mentioning the work of Hou et al. [13] and Hou et al. [14] in simulating fracture in asphalt mixtures using phase field approach. However, their model can only simulate brittle fracture therefore it is not suitable for simulating quasi-brittle behaviour of asphalt mixtures.

The present contribution therefore aims at exploring the appropriateness of the phase field model for damage modelling in asphalt mixture. In particular, phase field model coupled with cohesive zone approach is used to simulate quasi brittle behaviour of asphalt mixture. Such a numerical tool will help in an efficient design, prediction and understanding of damage in asphalt pavements under various conditions. The bulk material is modelled as a homogeneous solid. Damage in asphalt concrete is represented with the damage phase field variable. The inelastic material behaviour around the crack tip is modelled using a bi-linear traction separation law. In order to solely investigate the performance of the phase field model to simulate damage in asphalt mixtures, numerical test at a uniform low temperature of -10ºC is performed. Therefore, strain rate/temperature effects are not considered in this contribution.

The remainder of the paper is organized as follows. Section 2 presents the governing equations of the phase field model and its finite element discretization. Section 3 discusses the analysed model problem and implementational aspects of the phase field model in a finite element computer program. A single edge notch beam is numerically simulated to investigate the performance of the phase field model. Numerical results are validated against the experimental observations of Song et al. [2]. Discussion on the analysis results is given in section 4. Section 5 presents main conclusions drawn from the analysis.

2 Diffuse damage model for asphalt concrete

Consider a body with domain Ω with its external boundary denoted by $\partial\Omega$. The body is subjected to prescribed displacements $\acute{u}$ on the surface $\partial\Omega_u$ and prescribed tractions $\acute{t}$ on the surface $\partial\Omega_t$, figure 1. The domain Ω is also subjected to a body force b and contains an internal sharp crack *S*. Within the context of phase field method, the crack surface is regularized over the localization band B, such that, the sharp crack surface A_s is approximated with a diffuse functional A_d.

$$A_s = \int_B \delta_s dV \approx A_d = \int_B \gamma(d, \nabla d) dV \qquad (1)$$

in which γ is the crack surface density function approximating the Dirac-delta function δ_s of a sharp crack. A damage phase field d is defined over the domain Ω such that d=1 representing the fully damaged material and $d = 0$ represents the undamaged material. The damage phase field, d varies between 0 and 1 within the localization band B.

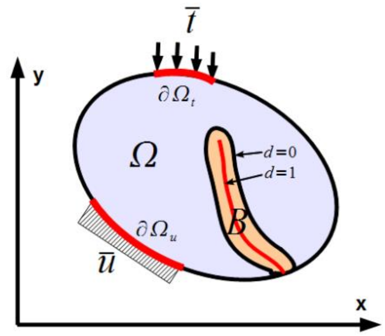

Figure 1: A solid body with a diffuse crack

2.1 *Governing equations*

The total potential Π of a system is defined as the sum of internal potential energy, fracture energy and external potential energy, mathematically given as

$$\Pi(u,d) = \int_\Omega \underbrace{\omega(\mathrm{d})\Psi_o}_{\text{strain energy density},\Psi} dV + \int_\Omega G_f \underbrace{\frac{1}{c_o}\left[\frac{1}{b}\alpha + b|\nabla d|^2\right]}_{crack\ surface\ density,\gamma} dV - \Pi^{ext} \quad (2)$$

G_f is the fracture energy, b is the length scaling parameter, α is the crack geometric function, Π^{ext} denotes the external potential energy, given as

$$\Pi^{ext} = \int_\Omega b \cdot u dV + \int_{\Gamma_t} \bar{\mathrm{t}} \cdot u dA \quad (3)$$

$\omega(d)$ is the degradation function which describes the degradation of elastically stored energy Ψ_o and possess the following property $\omega(0) = 1, \omega(1) = 0, \omega'(1) = 0$. The governing equations in weak form can be obtained by the minimization of the total potential (2), which yield the following coupled system of equations

$$\int_\Omega \left(\frac{\partial\Psi}{\partial\epsilon} : \nabla^{sym}\delta\mathrm{u}\right) dV + \int_\Omega b\delta\mathrm{udV} + \int_{\partial\Omega_t} \bar{\mathrm{t}}\delta\mathrm{udA} = 0 \quad (4)$$

$$\int_B \left(\omega'(\mathrm{d})\frac{\partial\Psi}{\partial\omega}\delta\mathrm{d}\right) \mathrm{dV} + \frac{\mathrm{G_f}}{\mathrm{c_o}}\left(\frac{1}{\mathrm{b}}\alpha'\delta\mathrm{d} + 2\mathrm{b}\nabla d \cdot \nabla\delta\mathrm{d}\right) \mathrm{dV} \leq 0 \quad (5)$$

In accordance with the weighted residual method, the (u, d) and $(\delta u, \delta d)$ are identified as test and trial functions respectively. The problem is now stated as: Find $u \in U_u$ and $d \in U_d$ such that equations (4) and (5) are satisfied. The test and trial spaces, (U_u, U_d) and (V_u, V_d)respectively, are defined as

$$U_\mathrm{u} \coloneqq \{\boldsymbol{u} | \boldsymbol{u} = \bar{\boldsymbol{u}} \quad \forall x \in \partial\Omega_u\}, \qquad V_\mathrm{u} \coloneqq \{\delta\boldsymbol{u} | \boldsymbol{\delta u} = \boldsymbol{0} \quad \forall x \in \partial\Omega_u\} \quad (6)$$

$$U_d \coloneqq \{d | \boldsymbol{d} \in [\boldsymbol{0}, \boldsymbol{1}], \quad \dot{\mathrm{d}}(\mathrm{x}) \geq 0 \forall x \in B\}, \qquad V_d \coloneqq \{\delta\mathrm{d} | \boldsymbol{\delta d} \geq \boldsymbol{0} \quad \forall x \in B\} \quad (7)$$

Equations (5) and (6) are supplemented with the constitutive relations. The relation between Cauchy stress σ and small strain ϵ is defined as $\sigma \coloneqq \frac{\partial \Psi}{\partial \epsilon} = \omega \frac{\partial \Psi_o}{\partial \epsilon}$. For linear elastic material, the stress can be defined as $\sigma \coloneqq \frac{\partial \Psi}{\partial \epsilon} = \omega(D\colon \epsilon) = \omega(D\colon \nabla^{sym} \delta u)$. Where D is the material elastic stiffness tensor. The damage driving force, Y, is defined as

$$Y \coloneqq -\frac{\partial \Psi}{\partial d} = -\omega' \acute{Y} \quad (8)$$

With $\acute{Y} = \frac{\partial \Psi}{\partial \omega}$ is the effective damage driving force. Equations (4) and (5) can now be written as

$$\int_{\Omega} (\boldsymbol{\sigma}\colon \nabla^{sym} \delta \mathrm{u})\, dV \;+\; \int_{\Omega} b \delta \mathrm{udV} + \int_{\partial \Omega_t} \bar{\mathrm{t}} \delta \mathrm{udA} \;=\; 0 \quad (9)$$

$$\int_{\mathrm{B}} (\omega'(\mathrm{d}) \bar{Y}\, \delta \mathrm{d}) \mathrm{dV} + \frac{\mathrm{G_f}}{\mathrm{c_o}} \left(\frac{1}{\mathrm{b}} \alpha' \delta \mathrm{d} + 2\mathrm{b} \nabla d \cdot \nabla \delta \mathrm{d} \right) \mathrm{dV} \leq\; 0 \quad (10)$$

2.2 Discretization of weak form

Considering a two dimensional (2D) numerical problem, the domain Ω is divided into n_e number of finite elements. The displacement and damage phase field are approximated as

$$u^h(x) = \sum_I N_I(x)\, a_I^u \;, \qquad d^h(x) = \sum_I N_I(x)\, a_I^d \quad (11)$$

In which N_I is the standard finite element matrix of shape functions for the node I. a_I^u and a_I^d are the nodal unknown displacement and damage phase field degrees of freedom (dofs), respectively. Accordingly, the strain field and the gradient of damage phase field is given as

$$\epsilon^h(x) = \sum_I \mathbf{B}_I^u(x)\, a_I^u \;=\; \mathbf{B}^{\mathrm{u}} \mathbf{a}^{\mathrm{u}}, \qquad \nabla \mathrm{d}^{\mathrm{h}}(x) = \sum_I \mathbf{B}_I^d(x)\, a_I^d \;=\; \mathbf{B}^{\mathrm{d}} \mathbf{a}^{\mathrm{d}} \quad (12)$$

Incorporating the approximations into the weak form equations (9) and (10), the following two discretized equations are obtained

$$R^u \;= \int_{\Omega} (\mathbf{N}^{\boldsymbol{u}})^T b \;\mathrm{dV} + \int_{\partial \Omega_{\mathrm{t}}} (\mathbf{N}^{\boldsymbol{u}})^T \bar{\mathrm{t}}\, \mathrm{dA} - \int_{\Omega} (\mathbf{B}^u)^T \boldsymbol{\sigma}\, dV = 0 \quad (13)$$

$$R^d \;= -\int_{\Omega} (\mathbf{N}^d)^T \left(\omega' \bar{\mathrm{Y}} + \frac{\mathrm{G_f}}{\mathrm{c_o b}} \alpha' \right) \mathrm{dV} - \int_{\Omega} (\mathbf{B}^d)^T \left(\frac{2b}{\mathrm{c_o}} \mathrm{G_f} \nabla \mathrm{d} \right) \mathrm{dV} \leq \mathbf{0} \quad (14)$$

The above equations are solved in a nonlinear setting using Newton-Raphson iterative scheme.

2.3 Failure criterion

The equation $\bar{Y} = \frac{\partial \Psi}{\partial \omega}$ gives similar fracture response both in compression and tension. Therefore, a modified effective crack driving force $\bar{Y}$ is used to simulate fracture in brittle/quasi-brittle materials. In this work it is assumed that fracture occurs when the local principal tensile stress exceeds the tensile strength of the material. Consequently, the following form of effective crack driving force is used

$$\bar{Y} = max \left(\frac{f_t^2}{2E_o}, max\, \overline{Y_n} \right), \qquad \overline{Y_n} = \frac{1}{2E_o} (\sigma_1)^2 \quad (15)$$

in which f_t is the tensile strength of the material and E_o is the modulus of elasticity

2.4 Degradation function

Following Wu [12], the following form of degradation function is adopted

$$\omega(d) = \frac{(1-d)^p}{(1-d)^p + a_1 d + a_1 a_2 d^2} \qquad (16)$$

The parameters $p = 2, a_1 = 4l_{cz}/\pi b, a_2 = -0.5$ are used to simulate bilinear traction-separation law. l_{cz} is the Irwin's internal length.

3 Methodology

The phase field model is implemented in an object-oriented C++ language. An open source JIVE library is used for the implementation of the finite element code (JIVE is an open source numerical toolkit for the solution of partial differential equations). Table 1 presents the flow of computations in a finite element code with phase field model.

Table 1-Flow of computations in a finite element code with phase field model

- Initialization: The displacement $\mathbf{a}_n^u$, damage phase field a_n^u and $\bar{Y}_n$ at time t_n are known
- For each loading step n to n+1
- Set $\left(\mathbf{a}_n^{u(0)}, a_n^{d(0)}, \bar{Y}_n^{(0)}\right)$ equals to $(\mathbf{a}_\mathbf{n}^\mathbf{u}, a_n^d, \bar{Y}_n)$ $and\ j = 1$
- For each iteration j
    - Calculate Cauchy stress: $\boldsymbol{\sigma}_{n+1}^{(j)}\left(\mathbf{a}_{n+1}^{u(\mathrm{j})}, a_{n+1}^{d(\mathrm{j})}\right)$
    - Calculate history variable: $hist_{n+1}^{(j)} = max\left(hist_n, \frac{\left(\sigma_{1(n+1)}^{(\mathrm{j})}\right)^2}{2E_o}\right)$
    - Calculate $\bar{Y}_n$: $\bar{Y}_{n+1}^{(j)} = \mathbf{max}\left(\frac{f_t^2}{2E_o}, hist_{n+1}^{(j)}\right)$
    - Calculate displacement and damage phase field $\left(\mathbf{a}_{n+1}^{u(\mathrm{j})}, a_{n+1}^{d(\mathrm{j})}\right)$ using equations (13) and (14)
    - Check convergence
- If converged then update variables: $(\mathbf{a}_\mathbf{n}^\mathbf{u}, a_n^d, \bar{Y}_n)$ equals to $\left(\mathbf{a}_{n+1}^{u(\mathrm{j})}, a_{n+1}^{d(\mathrm{j})}, \bar{Y}_{n+1}^{(\mathrm{j})}\right)$
- Go to next step

3.1 Single edge notch beam.

A single edge notch beam of Song et al. [2] is numerically simulated in this work. Song et al. [2] performed test on a single edge notch beam. The geometry of the beam is shown in Figure 2a. The beam contains an initial notch of length 19mm. In the experiment crack mouth opening displacement (CMOD) is increased at a linear rate for a stable mode-I crack growth. The beam is made of asphalt mixture consisting of a 9.5 mm nominal maximum aggregate size (NMAS) and a performance grade (PG) 64-22 asphalt binder. The experiment was performed at low temperature (-10ºC) to characterize the fracture behaviour at low temperature.

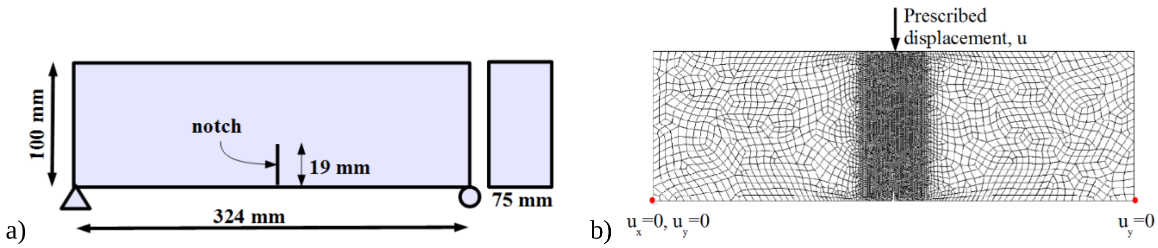

Figure 2: a. Geometry and boundary conditions of single edge notch beam and b. finite element mesh

Figure 2b shows the finite element mesh of the beam used in the numerical simulation. The beam is modelled with a two-dimensional, plane strain, 4-noded quadrilateral element. A minimum element size of 1 mm is used in the expected damage growth region whereas an element size of 5 mm is used in rest of the model. The notch width is taken as 2mm in the numerical model. Moreover, it is assumed that the bulk material essentially behaves elastically at low temperatures and therefore temperature effects are ignored. The bulk material is modelled as elastic, homogeneous material with modulus of elasticity E=14.2 GPa and Poisson's ratio ν = 0.3 [2]. The fracture properties used in the analysis are tensile strength f_t= 3.56MPa and fracture toughness G_f=0.344 J/m^2[2]. In the numerical simulation a downward displacement is applied in the middle of the top surface of the beam at a linear rate. The nonlinear finite element equations are solved using a Newton-Raphson iterative scheme with a tolerance of 1.0E-3.

4 Results and discussion

Figure 3a compares the load versus crack mouth opening displacement (P Vs CMOD) curve obtained from the present phase field model with that of experimental results. It is observed that the numerical result is in good agreement with that of experimental curve. Figure 3a also compares the P vs CMOD curve obtained by Song et al. [2] using a finite element analysis with interface elements. It is observed from the figure that the numerical result obtained by Song et al. [2] shows an initial complaint behaviour. This is due to the use of dummy stiffness in the interface formulation to simulate rigid interface before initiation of a failure. On the contrary, in the present phase field model the damage initiates once a failure criterion is met. Therefore, an initial high stiffness (portion of the curve before the peak load) is also well predicted by the present numerical model. Figure 3b shows the deformed shape of the beam representing mode-I fracture through an unstructured finite element mesh.

Figure 4 shows the damage growth in a single edge notch specimen at different levels of crack mouth opening displacements. It is observed from the figures that damage initiates at the notch tip before the peak load, figure 4a. After the formation of a macro crack (represented with a fully damaged zone $\omega = 1$) the load drops quickly which represents a fast crack growth. Moreover, the damaged zone ahead of the macro crack is large representing a large fracture process zone ahead of a crack tip compared to the damaged zone left and right sides of the macro crack. This represents a sharp crack as observed in the experiments of Song et al. [9]. As the load increases the damage region grow straight up to the top surface of the beam representing mode-I fracture. Additionally, it is observed that as the crack approaches the top surface of the beam the crack growth slows down this is also evident from the P Vs CMOD curve (figure 3a) where the load drop is more gradual in the later part of the curve.

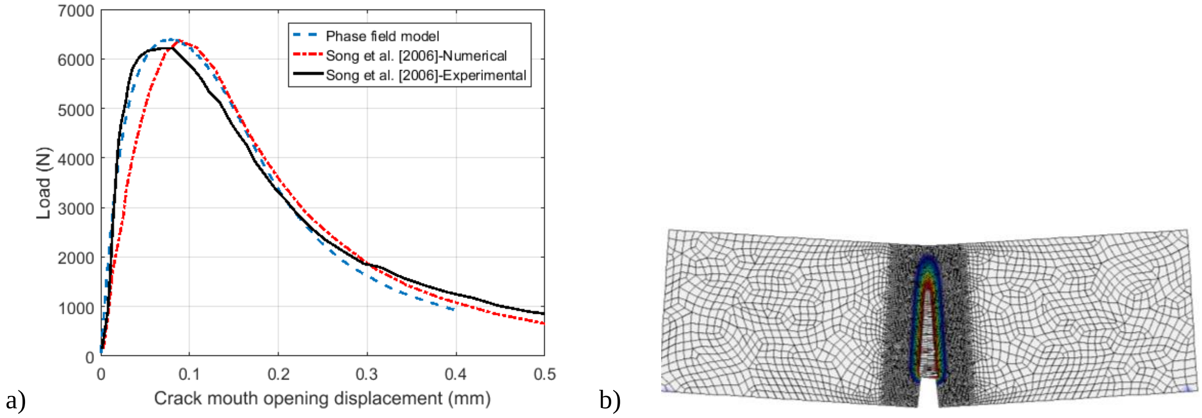

Figure 3: a) Comparison of Load Vs Crack mouth opening displacement curve with the experimental results, b) deformed shape

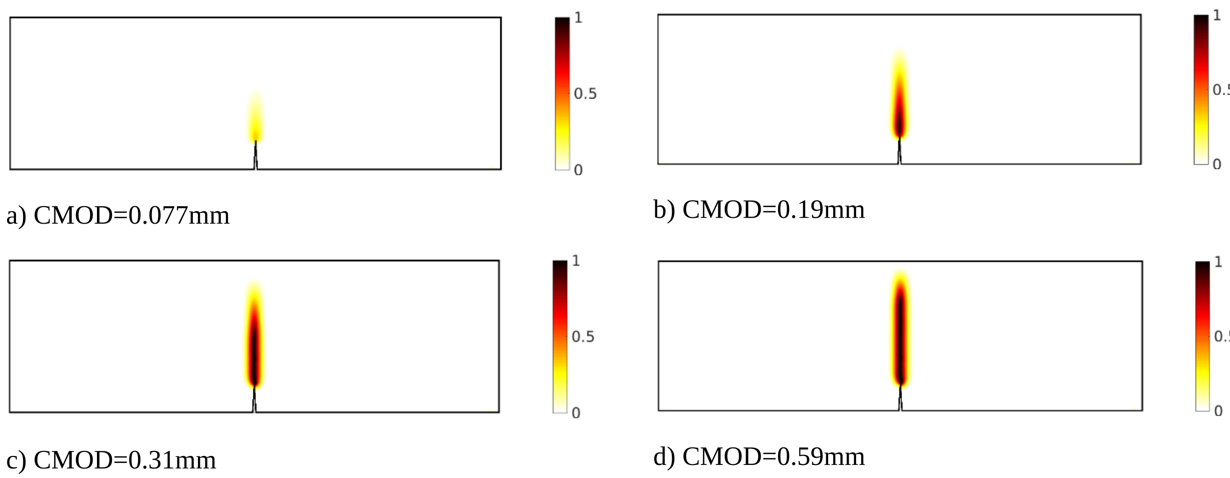

Figure 4: Damage growth at different instants of loading

5 Conclusion

In this contribution a phase field diffuse damage model is presented for the simulation of damage in asphalt concrete. A crack potential function regularized over a localization band is used to simulate sharp crack. A crack driving force integrated with the cohesive constitutive law is used to simulate nonlinear behaviour around the crack. A mode-I fracture in an asphalt concrete is numerically simulated and the results are compared with the experimental observations. It is observed that the presented model effectively and accurately simulated mode-I fracture in asphalt concrete at low temperature. The proposed model does not require the crack path to be known a priori as in the case of interface elements. Moreover, the model successfully simulated an initial stiff behaviour unlike interface element analysis which shows an initial complaint behaviour due to the use of dummy stiffness in its formulation. The phase field model does not require any special algorithm to track crack trajectories and can be easily implemented in a finite element code. At present the model is limited to simulate mode-I fracture without considering rate/temperature effects in asphalt concrete. Future work will focus on extending the model to the case of mixed mode cracking under various strain rates.

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